

Part A (15 points total)

Money

A rich mother decides to give some money to her 3 children, Ali, Betty and Charles.

She gives the money as follows. She gives RM 1 to Ali, then RM 2 to Betty, RM 3 to Charles, RM 4 to Ali, RM 5 to Betty and so on, until all the money has been distributed.

For example, if she starts with RM 12, then Ali will receive $RM\ 1 + RM\ 4 = RM\ 5$, Betty would receive $RM\ 2 + RM\ 2 = RM\ 4$, and Charles would receive RM 3. (Betty receives only RM 2 in the second round because by this point, RM 10 out of the RM 12 has been given. Charles receives nothing in the second round because there is no money left.)

We know that the rich mother has more than RM 12 to give. We also know that Betty received RM 28 in total.

1. (3 points) How much did Ali receive in total?

Stars

Beginning with some number n , you write a line of ‘*’s by repeatedly applying the following rules:

- If n is 0, stop.
- If n is odd, write a single ‘*’ and reduce n by 1.
- If n is even, divide n by 2.

For example, if you begin with $n = 3$ then you would proceed as follows. Since 3 is odd, you write a single ‘*’ and subtract 1 to give $n = 2$. Since 2 is even, you divide by two giving $n = 1$. Finally, since 1 is odd you write another ‘*’ and subtract one. Now $n = 0$ and you stop, having written 2 ‘*’s in total.

2. (3 points) If you begin with the number $n = 99$, how many ‘*’s do you write in total?

Buried Treasure

A treasure has been buried and 5 pirates (A, B, C, D and E) have partial instructions on where the treasure is located.

- A: 100 meters E, 60 meters N.
- B: 100 meters E, 60 meters S.
- C: 100 meters W, 60 meters N.
- D: 100 meters W, 40 meters N.
- E: 100 meters E, 40 meters N.

Unfortunately, if we combine all the 5 instructions together (that is, go 100m E, 60m N and then 100m E, 60m S, and so forth), we arrive at a wrong location.

The treasure is buried 200 meters E and 100 meters N. It can be found using 4 instructions instead of 5.

3. (3 points) Whose instructions should be excluded? A, B, C, D or E?

Last Stone Loser

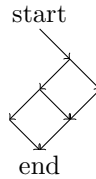
Two players play the following game. They start with a pile of 60 stones. They take turns to remove 1, 2, 3, 4, 5, or 6 stones per turn. The player who removes the last stone(s) loses.

4. (6 points) How many stones should the first player who starts, take on his first turn to guarantee that he wins?

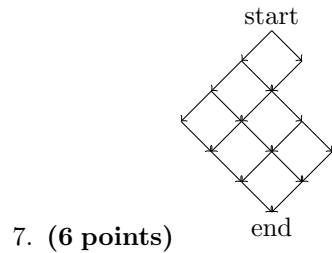
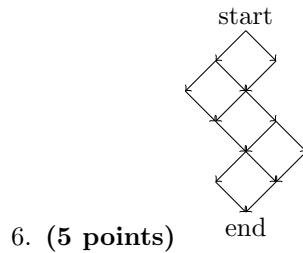
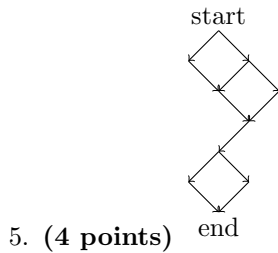
Part B (45 points total)

Number of Paths (15 points total)

We want to find the number of paths to get from start to end. We can only move down-left (L) or down-right (R). For example, in the diagram below there are a total of 3 paths: RRLL, RLRL, and RLLR.



For each of the diagrams below, how many paths are there to get from start to end?



Swapping Sort (15 points total)

You have a set of A, B and C letters arranged in a sequence. You want to sort them so that the A's are to the left, the B's are in the middle, and the C's are to the right.

The sorting process involves swapping pairs of letters. However, you may only swap two letters if they have *exactly* one letter between them. For example, if we have B A C A, you can swap B with C but you cannot swap B with either of the A's.

For example, B C A B needs a minimum of 2 swaps to sort in the correct order: B C A B → A C B B → A B B C.

For each of the following sequences, what is the *smallest* number of swaps needed to sort in the correct order?

- 8. (4 points) B C C B A A
- 9. (5 points) C B C B A C B A A C
- 10. (6 points) C B C B B A B A A C A C

Library Books (15 points total)

A library has several different books in a stack. Originally, they are in order 1 2 3 ... N with 1 on top. When a book is borrowed, the order does not change. For instance if there were 4 books ($N = 4$) and book 3 was borrowed, the stack would become 1 2 4.

Several books are borrowed, and a week later returned. When they are returned, they are put anywhere in the stack. In the example above, if book 3 was returned the stack could be 3 1 2 4, 1 3 2 4, or 1 2 4 3.

For each of the following questions, the order shown is the order in the stack after all the books have been returned. What is the *smallest* number of books that could have been borrowed?

- 11. (4 points) 3 5 7 6 2 1 4
- 12. (5 points) 2 5 3 7 4 6 1 8
- 13. (6 points) 5 6 3 1 9 2 7 8 4